

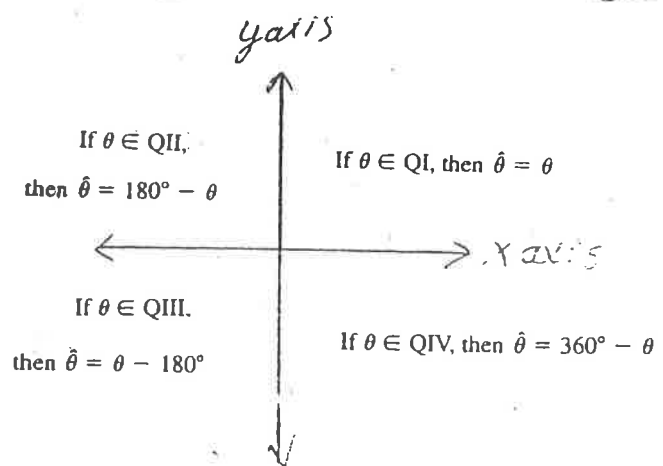
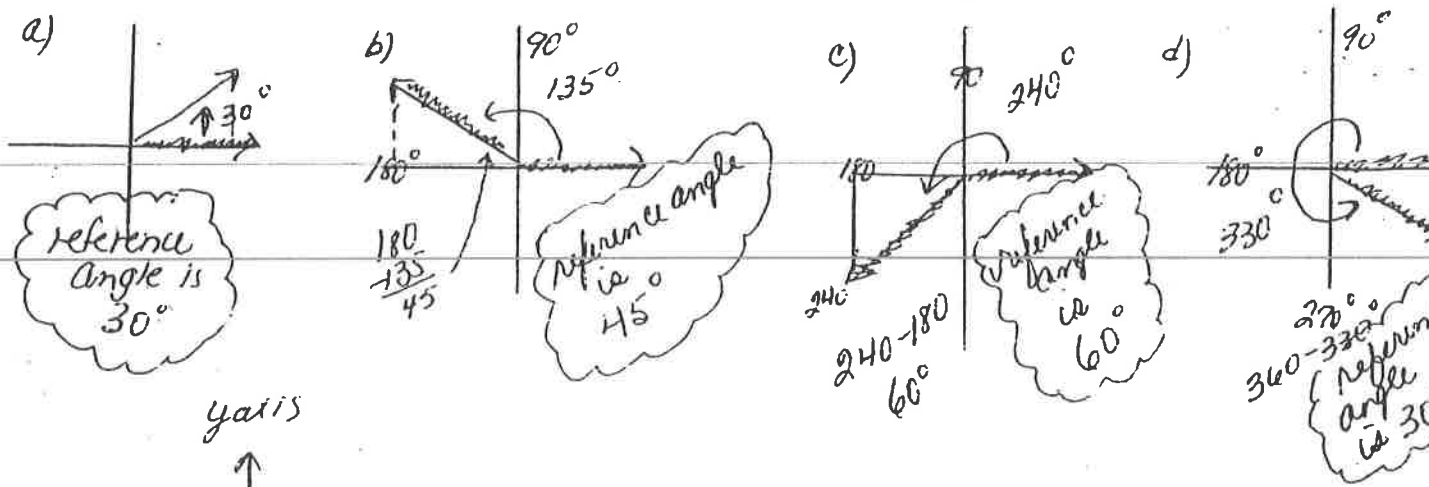
**DEFINITION**

The *reference angle* (sometimes called related angle) for any angle  $\theta$  in standard position is the positive acute angle between the terminal side of  $\theta$  and the  $x$ -axis. In this book, we will denote the reference angle for  $\theta$  by  $\hat{\theta}$ .

*\*  $\hat{\theta}$  is always positive and always between  $0^\circ$  and  $90^\circ$ .  
That is, a reference angle is always an acute angle.*

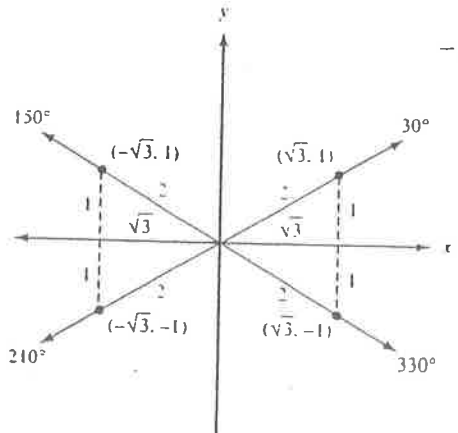
**EXAMPLE 1** Name the reference angle for each of the following angles.

- a.  $30^\circ$     b.  $135^\circ$     c.  $240^\circ$     d.  $330^\circ$



**REFERENCE ANGLE THEOREM**  
A trigonometric function of an angle and its reference angle differ at most in sign.

Figure 2



As you can see, any angle with a reference angle of  $30^\circ$  will have a sine of  $\frac{1}{2}$  or  $-\frac{1}{2}$ . The sign, + or -, will depend on the quadrant in which the angle terminates.

$$\begin{aligned} \sin 150^\circ &= \sin 30^\circ = \frac{1}{2} \\ \sin 210^\circ &= \sin 330^\circ = -\frac{1}{2} \end{aligned}$$

They differ in sign only.

Using this discussion as justification, we write the following steps used to find trigonometric functions of angles between  $0^\circ$  and  $360^\circ$ .

- Step 1. Find  $\hat{\theta}$ , the reference angle.
- Step 2. Determine the sign of the trigonometric function based on the quadrant in which  $\theta$  terminates.
- Step 3. Write the original trigonometric function of  $\theta$  in terms of the same trigonometric function of  $\hat{\theta}$ .
- Step 4. Find the trigonometric function of  $\hat{\theta}$ .

**EXAMPLE 2** Find the exact value of  $\sin 240^\circ$ .

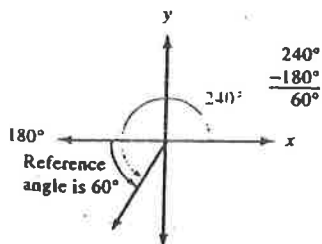
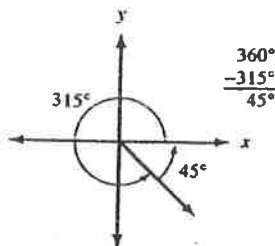


Figure 3

**EXAMPLE 3** Find the exact value of  $\tan 315^\circ$ .



**EXAMPLE 4** Find the exact value of  $\csc 300^\circ$ .

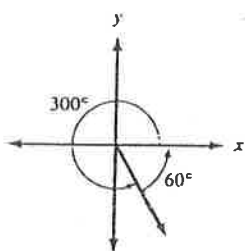


Figure 5

**EXAMPLE 5** Find the exact value of  $\cos 495^\circ$ .

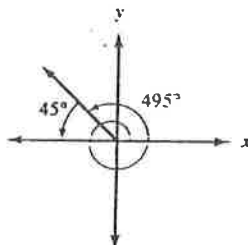


Figure 7

*Assign: # 124-125; # 2, 6, 9, 12, 11, 78 (every three problems)*

### Approximations

To find trigonometric functions of angles that do not lend themselves to exact values, we use a calculator. To find an approximation for  $\sin \theta$ ,  $\cos \theta$ , or  $\tan \theta$ , we simply enter the angle and press the appropriate key on the calculator. Check to see that you can obtain the following values for sine, cosine, and tangent of  $250^\circ$  and  $-160^\circ$  on your calculator. (These answers are rounded to the nearest ten-thousandth.)

$\sin 250^\circ = -0.9397$	$\sin (-160^\circ) = -0.3420$
$\cos 250^\circ = -0.3420$	$\cos (-160^\circ) = -0.9397$
$\tan 250^\circ = 2.7475$	$\tan (-160^\circ) = 0.3640$

To find  $\csc 250^\circ$ ,  $\sec 250^\circ$ , and  $\cot 250^\circ$ , we must use the reciprocals of  $\sin 250^\circ$ ,  $\cos 250^\circ$ , and  $\tan 250^\circ$ .

### Graphing Calculator

$$\csc 250^\circ = \frac{1}{\sin 250^\circ} = -1.0642 \quad | \quad \boxed{\frac{1}{\square}} \quad \boxed{\sin} \quad 250 \quad \boxed{\text{ENTER}}$$

$$\sec 250^\circ = \frac{1}{\cos 250^\circ} = -2.9238 \quad | \quad \boxed{\frac{1}{\square}} \quad \boxed{\cos} \quad 250 \quad \boxed{\text{ENTER}}$$

$$\cot 250^\circ = \frac{1}{\tan 250^\circ} = 0.3640 \quad | \quad \boxed{\frac{1}{\square}} \quad \boxed{\tan} \quad 250 \quad \boxed{\text{ENTER}}$$